ME 141
Engineering Mechanics

Portion 3
Equilibrium

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Condition of Equilibrium

➢ Recall Newton's Second law of Motion.

$$\sum F = ma$$
$$\sum M = I\alpha$$

➢ If a rigid body has no acceleration (linear and angular), that is either it's velocity(linear and angular) is zero (static) or it is moving with a constant velocity(linear and angular), then,

$$\sum F = 0, \sum M = 0$$

➢ These two equations are known as the condition of equilibrium.
➢ If the equations are expanded into their components in axial directions, then,

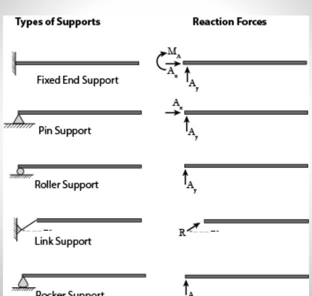
$$\sum \mathbf{F} = \sum F_x \mathbf{i} + \sum F_y \mathbf{j} + \sum F_z \mathbf{k} = 0 \qquad \sum \mathbf{M} = \sum M_x \mathbf{i} + \sum M_y \mathbf{j} + \sum M_z \mathbf{k} = 0$$
$$\sum F_x = 0 \qquad \sum M_x = 0$$
$$\sum F_y = 0 \qquad \sum M_y = 0$$
$$\sum F_z = 0 \qquad \sum M_z = 0$$

Equilibrium

2D Analysis | 3D Analysis

Equilibrium in 2D
Reactions at Supports and Connections

Equilibrium in 2D
Reactions at Supports and Connections



Condition of Equilibrium in 2D

- **In 3D,**

$$\sum \mathbf{F} = \sum F_x \mathbf{i} + \sum F_y \mathbf{j} + \sum F_z \mathbf{k} = 0 \qquad \sum \mathbf{M} = \sum M_x \mathbf{i} + \sum M_y \mathbf{j} + \sum M_z \mathbf{k} = 0$$
$$\sum F_x = 0 \qquad \sum M_x = 0$$
$$\sum F_y = 0 \qquad \sum M_y = 0$$
$$\sum F_z = 0 \qquad \sum M_z = 0$$

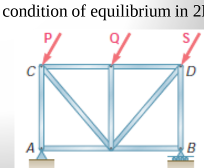
- **In 2D,**

$$\sum \mathbf{F} = \sum F_x \mathbf{i} + \sum F_y \mathbf{j} = 0 \qquad \sum \mathbf{M} = \sum M_z \mathbf{k} = 0$$
$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum M_z = \sum M_o = 0$$

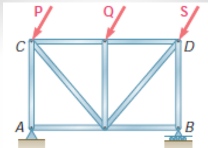
[0 is any point in the *x-y* plane or plane of the structure]

- These three equations are known as the condition of equilibrium in 2D.

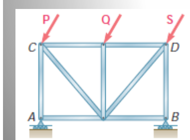
> • **These three equations are Sufficiently Essential to define a structure in equilibrium under any loading condition. The rigid body is then said to be fully or completely constrained.**
> • **Modification of these three equations can be possible.**



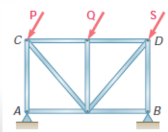
Condition of Equilibrium in 2D



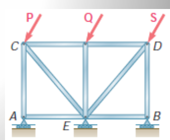
Fully Constrained



Partially Constrained

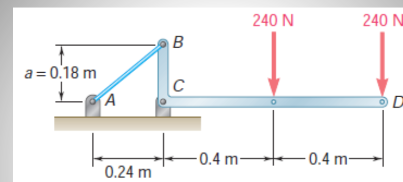


Statically Indeterminate



Improperly Constrained

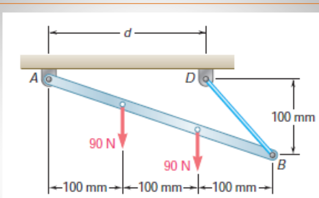
Problem 3.1 (Beer Johnston_10th edition_P4.15)



The bracket **BCD** is hinged at **C** and attached to a control cable at **B.** For the loading shown, determine (a) the tension in the cable, (b) the reaction at **C**.

Ans.: **Tx** = 1600 N ←, **Ty** = 1200 N ↓, T = 2 kN ; **C** = 2.32 kN ∠ 46.4°

Problem 3.2 (Beer Johnston_10th edition_P4.27)

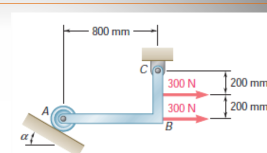


A rod **AB** hinged at **A** and attached at **B** to cable **BD** supports the loads shown. Knowing that d = 200 mm, determine (a) the tension in cable **BD**, (b) the reaction at **A**.

Ans.: **T** = 190.9 N ∠ 45°, **A** = 142.3 N ∠ 18.43°

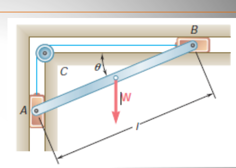
Problem 3.3 (Beer Johnston_10th edition_P4.21)

Determine the reactions at **A** and **C** when (a) α = 0°, (b) α = 30°.



Problem 3.4

A slender rod **AB**, of weight W = 30 N and length l = 1 m, is attached to blocks **A** and **B**, which move freely in the guides shown. The blocks are connected by an elastic cord that passes over a pulley at C. (a) Express the tension in the cord at the moment when θ = 30°.



Solution:

Steps:

1. Draw the Free Body Diagram of bar **AB.** **Assume tension in the cables are same.**
2. Apply three conditions of equilibrium, i.e.

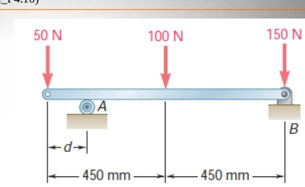
$$\overset{+}{\rightarrow}\sum F_x = 0,\ +\uparrow \sum F_y = 0,\ \circlearrowleft + \sum M_A = 0$$

3. Solve the three equations.

Ans.: T= 35.491 N

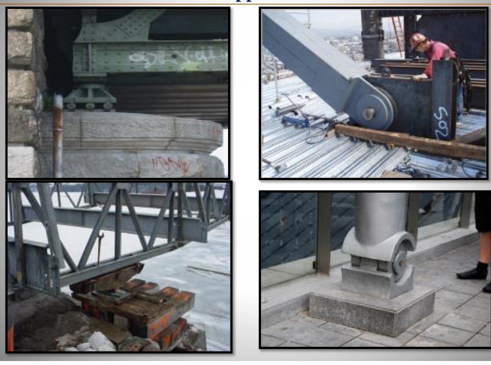
Problem 3.5 (Beer Johnston_10th edition_P4.10)

The maximum allowable value of each of the reactions is 180 N. Neglecting the weight of the beam, determine the range of the distance d for which the beam is safe.



Ans.: 150 mm ≤ d ≤ 400 mm

Equilibrium in 3D
Reactions at Supports and Connections



Equilibrium in 3D
Reactions at Supports and Connections

Equilibrium in 3D

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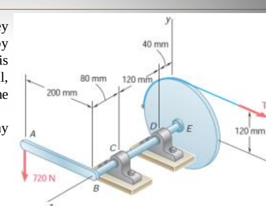
Problem 3.6 (Beer Johnston_10th edition_P4.91)

A 200-mm lever and a 240-mm-diameter pulley are welded to the axle **BE** that is supported by bearings at **C** and **D**. If a 720-N vertical load is applied at **A** when the lever is horizontal, determine (a) the Tension in the cord, (b) the reactions at **C** and **D**. Assume that the bearing at **D** does not exert any axial thrust.

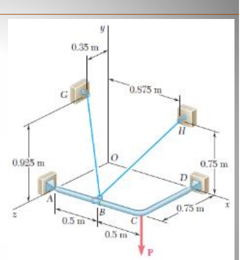


Ans.: T = 1.2 kN,
$\boldsymbol{C}$ = (0.4 kN)**i** + (1.2 kN)**j**,
$\boldsymbol{D}$= (-1.6 kN)**i** + (-0.48 kN)**j**

**Solve the problem assuming that the axle has been rotated at an angle 30 clockwise in its bearing and the 720N load is still vertical.

Problem 3.8 (Beer Johnston_10th edition_P4.138)

The frame **ACD** is supported by ball-and-socket joints at **A** and **D** and by a cable that passes through a ring at **B** and is attached to hooks at **G** and **H**. Knowing that the frame supports at point **C** a load of magnitude $\boldsymbol{P}$ = 268 N, determine the tension in the cable.



Solution:

Steps:
1. Draw the Free Body Diagram of bar **ACD.**
2. Apply moment equation about *AD.*

$$\sum M_{AD} = 0$$
$$\Rightarrow \lambda_{AD}\cdot(\mathbf{r}_{AB} \times \mathbf{T}_{BG}) + \lambda_{AD}\cdot(\mathbf{r}_{AB} \times \mathbf{T}_{BH}) + \lambda_{AD}\cdot(\mathbf{r}_{AC} \times \mathbf{P}) = 0$$

3. **Assume VALUE of tensions (*T*) in the cables are same.**

Ans.: T = 360 N

End of Portion 3

References

- **Vector Mechanics for Engineers: Statics and Dynamics**
Ferdinand Beer, Jr., E. Russell Johnston, David Mazurek, Phillip Cornwell.